M Prakash Institute **Solutions of IX Entrance Test 1**
17 November 2024, 9 to 11 am **Type AB**

All questions carry equal marks. *Total Marks 100*

1. How many three digited natural numbers are divisible by 17 ?

   Solution: The first three digit number divisible by 17 is $102 = 17 \times 6$. The last number is $17 \times 58 = 986$. So, total numbers$= 58 - 5 = 53$. **Ans. 53.**
2. 5 chairs and 3 tables cost Rs. 416 and 7 chairs and 5 tables cost Rs. 636. Then positive difference between cost of one chair and one table is?

   Solution: Suppose the cost of chair is x and that of table is y.
   So, we have $5x+3y = 416$ and $7x+5y = 636$. Solving these simultaneous equations, we get $x = 43, y = 67$ **Ans. 24.**
3. 6 coins of equal weight are melted and recasted into a single coin. The ratio of Gold and Silver in one coin was 2:1, 5:3 in two coins and 7:5 in remaining three coins. The ratio of Gold and Silver in the new recasted coin is $m : n$ where m, n are natural numbers having no common factor. Find $m + n$.

   Solution: Suppose the weight of each coin is 24. So, gold and silver is 16 and 8 in 1 coin, 15 and 9 in two coins and 14 and 10 in three coins. So total gold$= 88$ and silver$= 56$. So, ratio is $11 : 7$. **Ans. 18.**
4. If $a = \sqrt{6} + \sqrt{5}$ and $b = \sqrt{6} - \sqrt{5}$, Find value of $2a^2 - 5ab + 2b^2$.

   Solution: $2a^2 - 5ab + 2b^2 = (2a - b)(a - 2b) = (\sqrt{6} + 3\sqrt{5})(3\sqrt{5} - \sqrt{6}) = 39$
   Ans. 39.
5. Find the value of $\left[\sqrt{(8+\sqrt{28})} - \sqrt{(8-\sqrt{28})}\right]^2$

   Solution: $8 + \sqrt{28} = 7 + 1 + 2\sqrt{(7)(1)} = (\sqrt{7} + 1)^2$
   Similarly $8 - \sqrt{28} = 7 + 1 - 2\sqrt{(7)(1)} = (\sqrt{7} - 1)^2$.
   So, $\left[\sqrt{(8+\sqrt{28})} - \sqrt{(8-\sqrt{28})}\right]^2 = ((\sqrt{7}+1) - (\sqrt{7}-1))^2 = 4$ **Ans. 4.**
6. A has taken a loan of Rs. X from B . On first day A returns Rs. 1 to B, on second day A returns Rs. 2 to B, on third day A returns Rs. 3 to B and so on. After 30 days he returned 93% of the loan amount to B . How much is the remaining loan amount in rupees?

   Solution: A returns $1 + 2 + 3 + \cdots + 30 = 465$ rupees in 30 days. So, remaining amount is $\frac{7}{93} \times 455 = 35$ rupees. **Ans. 35.**
7. If $9^{20} + 9^{20} + 9^{20} = 3^x$, Find x.

   Solution: $9^{20} + 9^{20} + 9^{20} = 3(9^{20}) = 3((3^2)^{20}) = 3(3^{40}) = 3^{41}$ **Ans. 41.**
8. A tank is full of water and has 3 taps. One tap empties the tank in 8 hours. Second tap empties the tank in 10 hours. Third tap empties the tank in 40 hours. If all 3 taps opened simultaneously, after how many hours the tank will be empty?

   Solution: In hour, the first tank, the second tank and the third tank empties $\frac{1}{8}^{th}$, $\frac{1}{10}^{th}$, $\frac{1}{40}^{th}$ tank respectively. So, in one hour, $\frac{1}{8} + \frac{1}{10} + \frac{1}{40} = \frac{1}{4}^{th}$ tank is emptied. So, 4 hours are required to empty the tank. **Ans. 4.**

9. A had Rs. 600 in his pocket. Total number of Rs. 10 notes are double of Rs. 50 notes and number of Rs. 5 notes are more by 8 than the number of Rs. 10 notes. Find total number of notes.

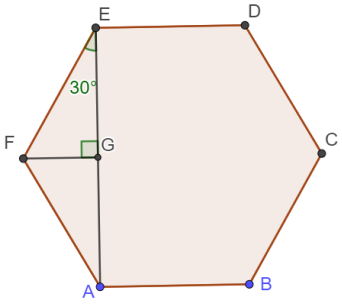
   Solution: Suppose number of Rs. 50 notes is x. So, number of Rs. 10 and Rs. 5 notes are $2x$ and $2x+8$. So, $50x+20x+5(2x+8) = 600 \Rightarrow x = 7$. So, total number of notes$= x + 2x + (2x + 8) = 5x + 8 = 43$. **Ans. 43.**

10. If P is the smallest positive integer that leaves remainder 1 when divided by all single digit positive integers, then find the positive square root of $(P - 21)$.

    Solution: $P = lcm(1, 2, 3, 4, 5, 6, 7, 8, 9) + 1 = 2521$. **Ans. 50.**

11. A regular hexagon has all 6 sides congruent and all 6 angles congruent. If $ABCDEF$ is a regular hexagon having each side length 6 , Find $\dfrac{AE^2}{2}$.

    Solution: As shown in the figure,
    $AE = 2(AG) = \sqrt{3}(EF) = 6\sqrt{3} \Rightarrow \dfrac{AE^2}{2} = \dfrac{(6\sqrt{3})^2}{2} = 54$
    Ans. 54.



12. Two articles were sold at Rs. 1485 each. One article incurred 10% loss while other incurred 10% profit. What is the total loss or profit in Rupees?

    Solution: Cost price of the article sold with loss$= 1485/0.9 = 1650$. Cost price of the article sold with profit$= 1485/1.1 = 1350$, so total cost price$= 1650 + 1350 = 3000$, so loss$= 3000 - 2(1485) = 30$ rupees. **Ans. 30.**

13. a, b, c and d are positive integers. $\dfrac{a}{b} = 2, \dfrac{b}{c} = 4, \dfrac{c}{d} = 8$ Then $\dfrac{4a + 2b + c + 2d}{a + 2b + 4c + 5d} = ?$

    Solution: $\dfrac{a}{b} = 2, \dfrac{b}{c} = 4, \dfrac{c}{d} = 8 \Rightarrow c = 8d, b = 4c = 32d, a = 2b = 64d$. So, $\dfrac{4a + 2b + c + 2d}{a + 2b + 4c + 5d} = \dfrac{4(64d) + 2(32d) + 8d + 2d}{64d + 64d + 32d + 5d} = 2$ **Ans. 2.**

14. On any day, from 12:05 AM to 11 : 55 PM, how many times the minute hand and hour hand of a clock coincide exactly over each other?

    Solution: Between 12.05 AM and 1am, the hands will not coincide. Then between every hour, they will coincide once till 12pm This is 11 times. Then again from 1pm, every hour once till 11.55 pm - this is 10 times. **Ans. 21.**

15. In the Wimbledon tennis tournament, total 128 tennis players participate. Each losing player goes out of the tournament and winning player advances to the next round. If total matches are played in the entire tournament is T, then find $T - 47$.

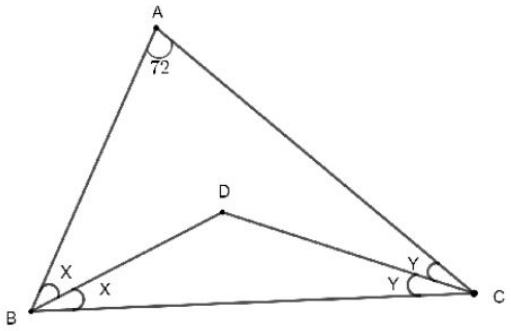
    Solution: Note that every match decides one loser. Selecting one winner is same as selecting 127 losers, so there are 127 matches. i.e. $T = 127 \Rightarrow T - 47 = 80$
    Ans. 80.

16. $834p$ is a four digited number with digit p at unit place. $q7$ is two digited number with q at tenth place . If $834p \times q7 = 55p383$, find $p + q$.

Solution: Since 3 is at unit place of the mutiplication, $p = 9$.
So, $q7 = 559383/8349 = 67$ **Ans. 15.**

17. In $\triangle ABC$, measure of angle A is 72 degree. Ray BD and ray CD are internal angle bisectors of $\angle B$ and $\angle C$ respectively. Find the value of (measure of $\angle BDC - 90°$).

Solution: $72+2x+2y = 180 \Rightarrow x+y = 54 \Rightarrow \angle BDC = 180-(x+y) = 180-54 = 126 \Rightarrow \angle BDC - 90 = 36$ **Ans. 36.**



18. If the permissible values of x satisfying the equation $(6+x)^{x^2-7x+12} = 1$ are p, q, r, find $p+q+r$.

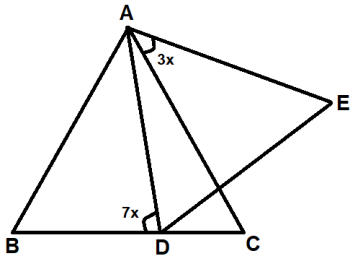
Solution: Either $6+x = 1$ or $x^2 - 7x + 12 = 0 \Rightarrow x = -5$ or 3 or 4 **Ans. 2.**

19. Simplify $\dfrac{\sqrt{1.96}+\sqrt{2.25}+\sqrt{2.89}}{(0.7)^2-0.03}$

Solution: $\dfrac{\sqrt{1.96}+\sqrt{2.25}+\sqrt{2.89}}{(0.7)^2-0.03} = \dfrac{1.4+1.5+1.7}{0.49-0.03} = \dfrac{4.6}{0.46} = 10$ **Ans. 10.**

20. $\triangle ABC$ and $\triangle ADE$ are equilateral triangles as shown with D is on $\overline{BC}$, $\angle EAC = 3x, \angle ADB = 7x$. Find x.

Solution:



$\angle BAD + \angle DAC = 60° = \angle DAC + 3x \Rightarrow \angle BAD = 3x$.
In $\triangle BAD, 3x + 7x = 120 \Rightarrow x = 12$. **Ans. 12.**